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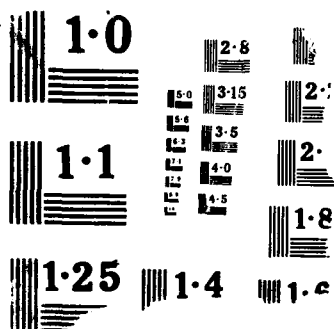
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Research problems considered included estimation with truncated data, burn in reliability problems, and accelerated life testings. Eighteen papers were published under this grant, with three more under preparation.

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From: Saul Blumenthal and Prem K. Goel, Ohio State University, OSU Research  
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The following research papers and reports were produced under AFOSR-84-0162.

RESEARCH OF SAUL BLUMENTHAL

I. ESTIMATION WITH TRUNCATED DATA

1. Asymptotic Expansions for Modified Maximum Likelihood Estimators with Percentile Truncated Data, Communications in Statist. Theor. Meth. 14, 1985, pp. 905-925.
2. Population or Sample Size Estimation, In Encyclopedia of Statistical Sciences Vol. VII, S. Kotz and N. L. Johnson (editors), 1986, pp. 100-110, J. Wiley, New York (with R. C. Dahiya).
3. Estimating N with Time Grouped, and Truncated Data from a Scale Parameter Distribution, in Communications in Statist. Theor. Meth. 16, 1987, pp. 1161-1179.
4. Estimation with Grouped Truncated Data, under revision for J. Amer. Statist. Assoc. (with R. C. Dahiya).
5. Estimating N with Percentile Grouped, and Truncated Data from a Scale Parameter Distribution, under revision for Communications in Statist. Theor. Meth.

We are given  $N$  random variables  $X_1, \dots, X_N$  with common density  $f(x; \theta)$  where both  $N$  and  $\theta$  are unknown. The values of the  $X$ 's can be observed only if they lie in the interval  $(0, T)$ . The problem addressed is estimation of  $N$  and  $\theta$ . Maximum likelihood estimators and variants of these, namely Bayes modal estimators are derived and asymptotic properties are studied.

In [3], an additional restriction is imposed, namely that the interval  $(0, T)$  is subdivided and only the number of  $X$ 's falling in each subinterval is reported, with all other details about actual values being lost. In addition to the problems mentioned above, questions of existence of the maximum likelihood estimators are addressed. In [4], for this same restriction, iterative computing schemes are examined, restrictions on the estimation procedures are derived to assure the stochastic consistency of the estimator of  $N$ , and Monte Carlo comparison of several estimators is carried out.

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In [1] and [5], instead of grouping at fixed time points and observability on an interval of length  $T$ , it is assumed that only a fixed subset of the quantiles of the  $X$ 's can be observed, the largest one being the  $(n/N)$ th, with  $n < N$ . Problems of the type considered in [3] and [4] are examined for this alternative model.

Paper [2] is a survey of work in this general area.

II. BURN-IN.

6. Burn-in with Mixed Populations, Ohio State University, Statistics Department  
Technical Report No. 372, August 1987 (with U.Q. Pan).

Closely related to the problem described above is the burn-in problem. In its simplest form, a batch of  $M$  items contains  $N$  (unknown) defectives with known failure distribution. A sequential stopping rule is desired for removing enough of the defectives to assure that with a specified high probability the reliability of the remaining items in the lot after burn-in exceeds some given lower bound. Various stopping rules are being studied for both exact and approximate (large lot size) properties. This technical report is based on the dissertation of Dr. Pan

III. RELIABILITY DEMONSTRATION.

7. Reliability Demonstration for New Series Systems, under revision for Technometrics.

8. A New Approximation for the Reliability of New Series Systems, in preparation.

Series systems exhibit wear out if the components tend to wear out as they age. Acceptance sampling criteria are usually stated in terms of Mean Time Between Failures (MTBF) for an aged (or equilibrium) system. However, the acceptance test is often administered to a newly produced system. If the test does not take into account the fact that the new system has a much larger MTBF than it will have after aging, too high a proportion of poor quality systems will be accepted. In [7], we have developed tests which take this aging phenomenon into account. They are based on a Poisson approximation for the distribution of the number of failures a new system will have in a fixed time period. In [8], we have studied alternative approximations and found that for moderate size systems, a binomial approximation fits the true distribution more closely than the Poisson or any of several other possible approximations. We are now looking at test design using the binomial.



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## RESEARCH OF PREM K. GOEL

1. Comments on "Computers in Statistical Research" by W. Eddy et al. *Statistical Science* 1 (1986) 444-445

The paper points out some of the salient points in the now famous Workshop on the use of Computers in Statistical Research, which was chaired by William Eddy. We stress the need for planning and for obtaining appropriate level of continuing support from the university sources for the dedicated computing facility, before applying to the federal agencies for equipment grant. It is also recommended that, in future, the Federal agencies provide more support for maintenance and upgrading of the computing equipment acquired under federal grant programs.

2. Some properties of the maximum likelihood strategy for pairing broken random samples, *Jour. of Statistical Planning & Inference* 16 (1987) (with T. Ramalingam), 237-248.

This paper studies the statistical properties of matching strategies for broken random samples from a bivariate population. In other words, a random sample of  $n$  pairs is drawn from a population, but the observed data consists of  $n$  observations from each of the two components separately. A maximum likelihood matching strategy is examined for its statistical properties. The proportion of approximately correct matches, based on this pairing strategy, is shown to converge in probability to a specified constant as  $n$  goes to infinity. This generalizes the main result in Yahav (1982), where this property was established for a very specific bivariate normal distribution. The small sample behavior of this proportion is studied via a Monte-Carlo simulation in the special case of bivariate normal parent population.

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3. Comparison of experiments & information in censored data. *Statistical Decision Theory and Related Topics IV, volume 2*, Eds. S. S. Gupta and James O. Berger (1988) 335-349 New York: Springer- Verlag.

In this paper, the concepts of comparison of experiments are reviewed first. It is shown that if one can construct an unbiased estimator of a parametric function based on the observations from an experiment  $E$  for which there is no unbiased estimator based on the observations from the experiment  $F$ , then  $E$  can not be sufficient for  $F$  in the sense of Blackwell. We then examine the class of information measures based on the  $f$ -divergence. This class of information measures contains several widely used information measures. If an experiment  $E$  is sufficient for the experiment  $F$ , then it is shown that, in this sense, the experiment  $E$  contains more information than the experiment  $F$ . We compare censored data experiments involving random right-censoring mechanisms based on two censoring variables  $Y_1$  &  $Y_2$ . It is shown that if  $Y_1$  is stochastically larger than  $Y_2$ , then the censored data experiment based on  $Y_1$  is sufficient for the censored data experiment based on  $Y_2$ . Thus the first experiment is more informative than the second for all information measures. This result generalizes some results in Hollander, Proschan, and Sconing(1985a, 1985b).

4. Bayesian design and analysis of accelerated life testing with step stress. *Accelerated Life Testing and Expert Opinion in Reliability: Proceedings of the International School of Physics "Enrico Fermi"* (1986) (eds. D.V. Lindley and C.A. Clarotti) Amsterdam: North Holland (with Morris H. DeGroot)

An accelerated life testing problem in which the stress  $s$  can take only a fixed, finite number of values is known as accelerated life test with step stress. In general, the item under test is started under the lowest stress first and if it has not failed until a certain specified time, then it is moved into the next level of stress. This process is continued until the item fails. Thus the life testy duration of the item is shortened. A Bayesian formulation of the problem is given in this paper. It is assumed that there are only two stress values under consideration (i) standard use environment condition (ii) a higher level of stress that is fixed in advance and is the same for all the items to be tested. However, the time at which an item on test is taken out of use environment and put under stress environment can be chosen by the experimenter subject to a cost structure. We

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consider the inference and the optimal design problem of when to change the stress as the test progresses.

5. Incorporating partial prior information : Ranges of Posterior Probabilities, Submitted Bayesian Inference and Decision Analysis: Volume in Honor of George Barnard, 1987 (with Mark Berliner).

This paper considers the incorporation of partial prior information concerning an unknown parameter in statistical inference and decision making problems. Such prior information is combined with experimental information to produce ranges of posterior probabilities concerning the parameter of interest. An example from Martz & Waller(1983), Bayesian Reliability book is pursued in depth.

6. Truncation, Information and the Coefficient of Variation. To appear Festschrift in Honor of Ingram Olkin 1988, Editors: Jim Press & Leon Jay Gleser (with Morris H. DeGroot and Maria J. Bayarri) New York: Springer-Verlag

The Fisher information in a selection sample from the truncated version of a distribution that belongs to an exponential family is compared with the Fisher information in a random sample from the untruncated distribution. Conditions under which there is more information in the selection sample are given. Examples involving the normal and gamma distributions with various selection sets, and the zero truncated binomial, Poisson and negative binomial distributions are discussed. A property pertaining to the coefficient of variation of certain discrete distributions on the non-negative integers is introduced and shown to be satisfied by all binomial, Poisson and negative binomial distributions. This work is being continued from the point of view of characterizing all distributions with this property.

7. Bayesian Software: Current Status and Additional Needs. Technical Report no. 366, Department of Statistics, Ohio State University. *To Appear* in Bayesian Statistics 3, Eds. D. Lindley, J. M. Bernardo, M.H. DeGroot A.F.M. Smith, Amsterdam: North-Holland

This paper provides a comprehensive information about the existing software for data analysis within the Bayesian paradigm. The paucity of programs seems to

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indicate that the Bayesian software available for a widespread use is still in its infancy. We have a long way to go before a general purpose Bayesian Statistical Analysis Package is made available. Alternatives for reaching this goal quickly are presented in the concluding section.

8. File merging methodology: Some statistical properties, Technical Report #333 (with T. Ramalingam), Department of Statistics, Ohio State University.

This report on *matching methodology* for merging micro data files to create larger files of data is a comprehensive survey and contains a variety of new results. Matching is often done to extract statistical information which cannot be obtained from the individual files that are incomplete. Current federal statistical practice involving multivariate file-merging techniques is typically not based on a formal statistical theory. In view of this situation, a survey on matching is given. All known models for matching are presented under a unified framework, which consists of three situations involving the same or similar individuals.

The properties of maximum likelihood strategy to match files of data involving the same individuals are derived via ranks and order statistics from bivariate populations. In addition, the properties of this strategy have been examined with respect to a more reasonable criterion called *epsilon-correct matching*. Asymptotic results for such situations, including (i) the Poisson approximation for the distribution of the number of correct matches, and (ii) convergence in probability of the average number of epsilon-correct matches, have been derived. Small sample properties, like the monotone behavior of the expected number of matches with respect to the dependence parameter of the underlying models, have also been proved.

Two matching strategies due to Kadane(1978) and one strategy due to Sims(1978) for merging files of data on similar individuals are discussed. These strategies are evaluated via a Monte-Carlo study of matching models involving trivariate normal distributions. This report has recently undergone a thorough revision. It will appear as a monograph in the Springer-Verlag Lecture Notes Series during 1988.

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9. Comparison of Selection Experiments, Technical Report No. 389, Department of Statistics, Ohio State University (with Morris H. DeGroot)

In this paper, the concepts of comparison of experiments in the context of comparing censored data experiments involving random right-censoring mechanisms based on two censoring variables  $Y_1$  &  $Y_2$  are review first. These results were developed in Goel(1987). However, in the experiments involving *selection models*, it is assumed that the observation  $Y$  is restricted to a specified subset of the sample space and the inference about  $\theta$  is to be based on a random sample from the density:

$$g(y, \theta) = f(y, \theta) / [\Pr\{X \in S | \theta\}].$$

In Bayarri and DeGroot(1986b), the information in selection models has been compared with the information in the unrestricted observations for various distributions and a variety of selection sets. This comparison is primarily in terms of Fisher information for the distributions belonging to the natural parameter exponential family and the set  $S = [\tau, \infty)$ . It is shown that, in general, the unrestricted experiment  $E$  has more Fisher information than the restricted experiment  $F$ . One exception to this result is the gamma distribution with shape parameter  $\alpha < 1$  for which the converse is true.

In contrast to the random right-censored data experiments, usually the experiment  $E$  has smaller risk than the experiment  $F$  for some decision problems and/or prior distributions whereas the reverse holds for other decision problems and/or prior distributions. Thus it is possible that the selection experiment is better than the original experiment for some decision problems and the reverse is true for some other decision problems, i.e., two experiments are not comparable. To obtain more meaningful ordering, Lehmann [1986] calls 'the experiment  $E$  *more effective* than  $F$ ' if for some interesting subclass of decision problems, the experiment  $E$  provides decision procedures with smaller risk than that based on the experiment  $F$ . An useful subclass of decision problems is that of test of hypotheses, for which the concept of *pairwise sufficiency*, denoted by  $E \geq_2 F$ , is relevant.

Clearly, if  $E \geq F$ , then  $E \geq_2 F$ , but the converse is not true. Furthermore, if the Fisher information exists, then  $E \geq_2 F$  implies that the experiment  $E$  has more Kullback-Leibler information than the experiment  $F$ , which in turn implies that  $E$  has more Fisher information than  $F$ , denoted by  $E \geq_F F$ .

Bayarri and DeGroot (1986b) show that for selection models in case of observations from a normal distribution with mean  $\theta$ , the unrestricted experiment is pairwise sufficient for the restricted experiment. Furthermore, they also show that for

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Binomial distribution with  $n=2$ , the unrestricted experiment is pairwise sufficient for the corresponding experiment with 0-class missing. In this paper, we mainly examine the pairwise sufficiency for comparing selection models for the class of distributions with monotone likelihood ratio (mlr) property and the selection set  $S(\tau) = [\tau, \infty)$ . In addition, some interesting results about the hazard rate for distributions with mlr and PF2 property have also been derived.

10. On the Stochastic Modelling of Fatigue Crack Propagation (with Panickos Palettas) In preparation.

An accurate assessment of Fatigue Crack Propagation (FCP) is important to the structural design of fracture critical aerospace components. Traditionally, the Paris-Erdogan model is devoted to modeling the variability in  $N(a)$  or the crack growth rate  $dN/da$  for fixed crack length  $a$ . Some stochastic models for FCP based on embedded Markov chains have also been developed. A review of the important concepts is presented.

An important problem in the modeling of FCP phenomena is to predict the number of load cycles  $N(a)$  required for the crack to grow to a specified length  $a$ . Two predictive models for FCP are examined in our work. The first is a simple predictive model based on Time Transformation (T T) ideas. Since individual specimen's crack growth curve and the mean curve can be considered to be from the same family of curves, the idea of T T has turned out to be an excellent way of predicting  $N(a)$ . The second model is based on Gamma Processes, in which  $N(a)$  is represented as a sum of a countable number of jumps of random height at a countable number of random points. A Pareto family of priors for the jump points, is suggested by the Paris-Erdogan model. Hierarchical Bayes is used to update the model parameters.

**In addition, the following papers are under preparation:**

(i) Characterization of independence based on the density of a linear combination of random variables.

(ii) Equivalence of OLS and BLUE of fixed effects in a Round Robin Design

(iii) Comparison of Censored and Selection Experiments To Appear in the Ohio State-Korea University Conference Proceedings.

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